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Developments in Operational Shelf Sea Modelling in Danish Waters

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An initial implementation of an operational system for the Danish waters has been carried out. The system consists of an observational network of water level stations, a two-dimensional model that computes simultaneously areas with different grid resolution, and a sequential data assimilation method. For assimilation of water level measurements an approximate Kalman filter algorithm, the ensemble Kalman filter, has been implemented. The ensemble Kalman filter has a computational cost much lower than the cost associated with a full Kalman filter application, and therefore it is suitable to be applied in an operational system. The error covariance matrix in this specific case tends to a quasi-steady state after a few days of assimilation. Thus, a Kalman filter with a constant weighting matrix has been applied during a 13-day test period to assimilate observed water levels in a model covering the entire North Sea and Baltic Sea area. The corrections achieved by the assimilation procedure are significant in most of the validation stations. Moreover, the error estimation provided by the filter is very accurate, especially in the inner Danish waters.

Keywords: operational model; nested two-dimensional model; data assimilation; Kalman filter; constant weighting matrix

Introduction

One of the most important phenomena that occurs in the regional shelf areas is the storm surge. The storm surge consists of a meteorological long wave motion, which produces an elevation of the water surface far above the level caused by the astronomical waves. The storm surge is a combination of two actions: extreme wind stress and reduced atmospheric pressure on the shallow coastal shelf areas (Bode & Hardy, 1997). The effects of a storm surge on the coastal areas can be devastating. Therefore, it is necessary to have the tools to predict the storm surge so far ahead in time that appropriate measures can be taken. The combination of numerical weather prediction models and hydrodynamic models forms the main frame of an operational storm surge forecast system (Bode & Hardy, 1997). The hydrodynamic model uses the predicted meteorological wind and pressure data to provide a prediction of the water level field. In the North Sea and Baltic Sea regions different numerical models have been running operationally during the last few years for storm surge prediction. The Dutch Continental Shelf model (Gerritsen *et al.*, 1995) is used for on-line estimation and forecasting of the tide and surge on the Northwest European Continental Shelf. For a similar purpose, the Danish Meteorological Institute runs an operational model of the North Sea and the Baltic Sea (Vested *et al.*, 1995) using a nested model with three levels of resolution. Another operational model in the region is the High Resolution Operational Model of the Baltic Sea (HIROMB) which runs operationally at the Swedish Meteorological Institute (SMHI) and consists of a three-dimensional baroclinic model (Funkquist, 1993).

The operational system should integrate a numerical model with the available observations using a data assimilation method. The best estimation of the state of the system can be obtained as a combination of the model's deterministic prediction and the observed variables. Some of the aforementioned operational models have already integrated data assimilation methods for assimilation of various variables such as water levels, currents and sea surface temperature.

This work presents the initial steps towards a new development of an operational system of the North Sea and the Baltic Sea. Although the operational system will be developed using a three-dimensional model, this work presents a test case based on a two-dimensional shallow water equation model. The next section gives a general description of the different elements of an operational system: the observational network, the numerical model and the data

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assimilation method. In the subsequent sections the three elements for the specific case of the North Sea and Baltic Sea operational model are described. The performance of the operational model during a 13-day test period in October 1997 is presented. In the final section some conclusions and recommendations for future developments in the operational system are given.

Elements of an operational system

A regional and shelf sea forecasting system consists of three main parts: an observational network, a dynamical model and a scheme used to assimilate data into the model (Robinson *et al.*, 1996).

Observational network

The number of available measurements of the ocean and seas has experienced an extraordinary increase during the past years. Observations of a variety of variables, including physical variables, such as water levels, velocity, temperature and salinity, chemical variables, such as nutrients and oxygen concentration and biological variables, such as phytoplankton, are nowadays available. Continuous measurements are necessary in an operational modelling system. This can be achieved by using moored instruments with ADCP, XBT and elevation sensors. Providing continuous data is the great advantage of these kinds of instruments but they have the disadvantage of covering only one single point in the horizontal. Remote sensing satellite data has the advantage of covering a large area with a fine spatial resolution but it has the disadvantage that the data is very sparse in time and covers only a thin band near the surface. In order to measure three-dimensional fields, instruments of the first type have to be employed.

The observational network of the Dutch Continental Shelf model that runs operationally at the Dutch Meteorological Institute (KNMI) (Gerritsen *et al.*, 1995) consists of satellite altimeter data and a network of water level stations. In the East Coast Ocean Forecast System (ECOFS) presented by Aikman *et al.* (1996) the observational network consists of water levels stations, and satellite data such as Sea Surface Temperature (SST) and Sea surface Height (SSH).

Dynamical models

Regarding dynamical models, generally the present state-of-art operational storm surge prediction schemes use two-dimensional shallow water equation models (Vested *et al.*, 1995; Gerritsen *et al.*, 1995). In shallow seas these type of models still provide a very good simulation of the water levels and they have very good forecasting capabilities. Three-dimensional multivariate models have to be used either when the two-dimensional description is insufficient in the region of interest or when it is necessary to model the vertical distribution as well as other physical (temperature and salinity), chemical and biological variables. An example of a pre-operational 3D modelling scheme can be found in Davies *et al.* (1998).

The application of nested grids in coastal areas is a very useful technique (Vested *et al.*, 1995). The use of nested areas allows the definition of high-resolution areas wherever is required in the model, maintaining a coarser resolution in other model areas with the consequent saving in computer requirements. The coarsest or regional grid will provide the boundary data for the fine resolution areas. In the coastal areas the fine resolution grid should provide a more detailed description of the ongoing processes.

Data assimilation methods

The adaptation of the most advanced data assimilation schemes, usually obtained from previous experiences in meteorology and oceanography, to the regional models is nowadays possible. It is well known that models contain errors generated mainly by errors in the forcing terms (boundary conditions and meteorological forcing terms), the model parameters and the poorly described or neglected physical processes in the system equations as well as mathematical approximations (for example unresolved sub-grid-scale motions). In order to take these errors into account, and hence obtaining a more realistic state of the system as well as a more reliable and accurate forecast, data assimilation has to be applied. A brief review of data assimilation techniques applied in regional coastal models is given below. For a more detailed review of data assimilation in meteorology and oceanography the reader is referred to the paper of Ghil and Malanotte-Rizzoli (1991).

In general, data assimilation techniques can be divided into two main groups, respectively, sequential and variational methods. In sequential data assimilation the model is integrated over the time interval producing a predicted system state, and whenever observations are available this state is updated or corrected using the new observations. Within this group of methods, *optimal interpolation*, most commonly used for numerical weather prediction (for example Lorenc, 1981 and Daley, 1991), is very useful when dealing with large sets of observations.

The method, however, is based only on statistical assumptions and not on the model dynamics. Another method, *nudging or newtonian relaxation*, consists of a dynamical model relaxation towards the observations. Sokolov *et al.* (1997) applied this method for assimilation of temperature and salinity data in the transport equations of a 3D baroclinic hydrodynamic model of the Baltic Sea.

Techniques based on the *Kalman filter* (Kalman, 1960) provide not only a corrected state of the system but also information about the system errors. The Kalman filter in its original form, however, is infeasible in large systems due to the huge computational requirements for propagation of the errors. The so-called suboptimal schemes (Todling & Cohn, 1994) are based on either a simplification of the model dynamics for propagation of the errors (e.g. Dee, 1991; Fukimori & Malanotte-Rizzoli, 1995; Cohn & Todling, 1996) or an approximation of the error covariance matrix. Kalman filter applications in regional coastal models have mainly been based on the latter approach. Heemink (1986) applied a steady (time invariant) Kalman filter where the Kalman gain can be calculated off-line, implying a significant reduction of the computational burden. This technique has been successfully applied for two-dimensional linear storm surge forecasting models in Heemink (1990), Vested *et al.* (1995), and Heemink *et al.* (1997). In order to use a time invariant Kalman filter, an efficient scheme based on a reduced rank approximation of the error covariance matrix has been introduced (Verlaan & Heemink, 1995; Cohn & Todling, 1996; Verlan, 1998). Examples of application of this technique, the reduced rank square root filter (RRSQRT), to 2D shallow water models can be found in Cañizares *et al.* (1998) and Heemink *et al.* (1997). A different method proposed by Evensen (1994) is the *ensemble Kalman filter*, which is based on a Monte Carlo simulation approach for propagation of the error covariance. An application of this method in a quasi-geostrophic ocean model for assimilation of altimeter data can be found in Evensen and van Leeuwen (1996). Madsen and Cañizares (1999) applied the ensemble Kalman filter in a two-dimensional shallow water model.

The second group of data assimilation methods are based on the variational principle. These methods can be considered more as smoothing procedures. Variational assimilation adjusts the model solution over the entire assimilation period to all the observations available in that period. In the area of coastal modelling, the variational method has been applied for off-line estimation of model parameters (Lardner *et al.*, 1993; Verlaan, 1994).

A preliminary implementation towards an operational model of the Danish waters

The Danish Meteorological Institute runs operationally a model covering the Danish waters. The set-up of that model was introduced in Vested *et al.* (1995) and it is based on a two-dimensional shallow water model using fine grid resolution areas to have a better description of the Danish waters. For these preliminary tests, a two-dimensional version of the DYNOCS (Dynamics of connecting seas) (Jensen, 1997) model has been used. Some results in a simplified version of this model have been presented in Cañizares and Madsen (1998) and Cañizares (1999). The following sections contain a description of the numerical model used for the tests and a description of the applied data assimilation method. The observational network is described together with the description of the model set-up.

The numerical model

The data assimilation method has been applied in the present study to the hydrodynamic module of the MIKE 21 modelling system, which solves the vertically integrated equations of continuity and conservation of momentum in two horizontal directions (see Abbott *et al.*, 1981; Vested *et al.*, 1995). MIKE 21 uses a finite difference approximation to solve the partial differential equations where the variables are defined on a space staggered rectangular grid with elevations at grid points and fluxes midway between grid points (Leendertse, 1964). A time-centred alternating direction implicit (ADI) scheme is adopted. The equations are solved in one-dimensional sweeps, alternating between x and y directions.

The MIKE 21 model allows the simultaneous use of areas with different grid sizes. The model uses a two-way nesting technique that ensures a dynamically consistent exchange of mass and momentum between the modelling grids of different resolution. The implementation uses a constant factor of 3 as a jump in resolution between the computational grids.

Data assimilation techniques implemented in MIKE 21

Sequential data assimilation procedures based on the Kalman filter (KF) have been implemented into the MIKE 21 model for assimilation of tide gauge measurements. The KF provides a successive correction of the state of the system as well as an estimate of the corresponding uncertainty by taking into account the inherent uncertainties, including errors in the open boundary conditions, errors in the meteorological forcing, and measurement errors.

For implementation of the Kalman filter in MIKE 21, the numerical model has to be formulated in a state-space form. The state variables to be considered are surface elevations and depth averaged velocities in the x and y-directions at every point of the horizontal grid. The numerical scheme in MIKE 21 can be written in state vector form as

$$x_k = f(x_{k-1}, u_k) \tag{1}$$

where x_k is the state vector, and u_k is the forcing of the system in terms of the surface elevations at open boundaries, and the meteorological forcing components in the momentum equations (wind stress and pressure gradient).

For modelling the uncertainty of the system, it is assumed that model errors are mainly related to errors in the forcing terms. At open boundaries, the tidal component of the surface elevation can usually be obtained with a relatively high accuracy, whereas the variations due to the meteorological effects may contain large errors (for example due to generation of a surge outside the model domain). The errors in the meteorological forcing terms are partly caused by uncertainties of the meteorological observations, and partly related to the physical description of the wind stress component and determination of the wind friction factor.

The error processes are assumed to be less spatially variable than the water flow process (Heemink, 1990), and the discrete error processes can thus be defined on a grid G2 that is coarser than the model grid G1 (in the case of a nested model G1 contains a set of areas with different grid sizes). A stochastic representation of the system Equation (1) can then be written

$$x_k = f(x_{k-1}, u_k + \Lambda \varepsilon_k) \tag{2}$$

where ε_k contains the model error at every grid point of G2, and Λ is a matrix that represents the sequence of linear interpolations between G2 and G1. The model error process is assumed unbiased, and the error statistics (variance structure as well as spatial and temporal correlation structure) are assumed known.

Measurements z_k of the state of the system are assumed to be available at certain points in the model grid G1. Measurement errors are partly related to the measurement equipment, and partly related to the uncertainty caused by the use of point measurements to represent grid averages. The stochastic representation of the measurement equation reads

$$z_k = C_k x_k + \eta_k \tag{3}$$

where C_k is a matrix that describes the relation between measurements and state variables, and η_k is a random measurement error with zero mean and known covariance matrix R_k.

Now, denote by x_k^f a one-step ahead forecast of the state of the system, according to the model operator f, cf. (1). The error covariance matrix P_k^f describes the uncertainty of this forecast. If measurements are available, cf. (3), the model forecast and the measurements can be combined to obtain an updated estimate of the state of the system. The Kalman filter update of the state vector and the error covariance matrix are given by

$$x_k^a = x_k^f + K_k(z_k - C_k x_k^f) \tag{4}$$

$$P_k^a = P_k^f - K_k C_k P_k^f \tag{5}$$

where K_k is the Kalman gain matrix

$$K_k = P_k^f C_k^T [C_k P_k^f C_k^T + R_k]^{-1} \tag{6}$$

which serves as a weighting function of model forecast and measurements and depends on the associated errors P_k^f and R_k. In (4)–(6) superscripts f and a refer to, respectively, forecast and analysis (or update), and superscript T indicates the transpose of a matrix.

For large systems, the propagation of the errors in the Kalman filter is the main bottleneck, imposing an unacceptable computational burden. Let n denote the dimension of the state vector (in the order 10^3–10^5 in models of realistic complexity), the propagation of the error covariance matrix requires $2n$ as much computing effort as is required to advance the deterministic model. The Kalman filter algorithm described below, the ensemble Kalman filter, provides an approximation of the error covariance propagation that significantly reduces the computational burden.

In the ensemble Kalman filter (EnKF) (Evensen, 1994), the statistical properties of the state vector are represented by an ensemble of possible state vectors. Each of these vectors is propagated according to the dynamical system subjected to model errors, and the resulting ensemble then provides estimates of the forecast state vector and the error covariance matrix. In the measurement update, the Kalman gain matrix is applied for each of the forecast state vectors. To account for measurement errors, the measurements are represented by an ensemble of possible measurements (Burgers *et al.*, 1998). The resulting updated sample provides estimates of the updated state vector and the error covariance matrix. Usually the ensemble size is much smaller than the dimension of the state vector, implying a significant reduction in computing

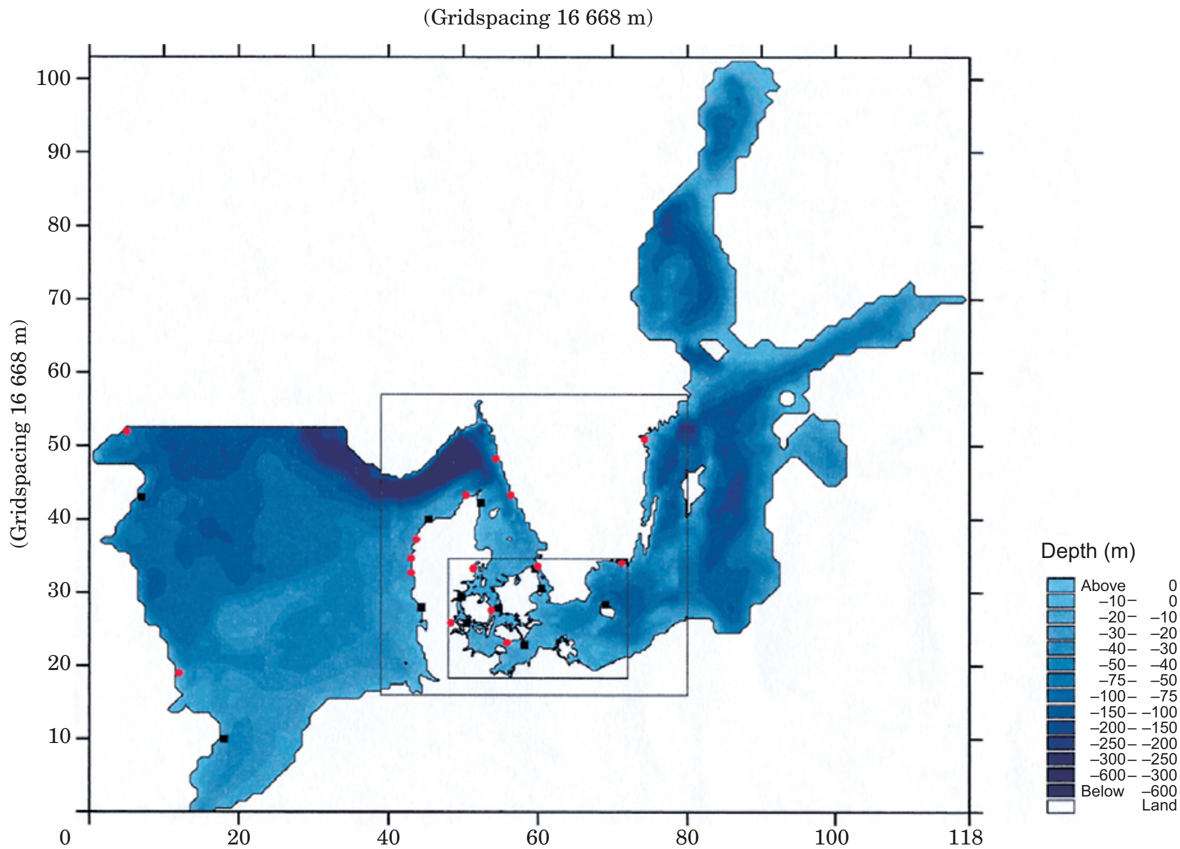

FIGURE 1. North Sea and Baltic Sea model bathymetry (depth in metres) with internal nested areas. Water level stations are represented with black squares (measurements) and red circles (validation).

time as compared to the full Kalman filter. For uncorrelated measurement errors, a sequential updating scheme based on Maybeck (1979) can be applied. This algorithm avoids the expensive calculation and storage of P_k^f as well as the matrix inversion in (6) for the calculation of the Kalman gain. In this implementation, scalar measurements are assimilated under the assumption that they are uncorrelated and therefore can be processed independently. A detailed description of the implementation of the ensemble Kalman filter in MIKE 21 can be found in Madsen and Cañizares (1999).

Different tests of data assimilation applications in two-dimensional models of the North Sea and North Sea and Baltic Sea have shown that the error covariance matrix becomes nearly invariant after 1–2 days of simulation. In order to make use of this feature a constant Kalman gain matrix can be calculated as an average of the nearly invariant Kalman gain calculated during 1–2 days of simulation. The data assimilation technique applied in this case will be similar to an optimal interpolation method, although in this case the constant weighting matrix has been calculated from a time variant sequential data assimilation method. Once the Kalman gain has been calculated (off-line calculation) it is not necessary to propagate the error covariance matrix, which, as it was previously mentioned, is the most costly part of the data assimilation method. Thus, application of the Kalman filter using a constant Kalman gain is only slightly more expensive than a model run without data assimilation

The observational network and the regional model set-up

The DYNOCS model domain is defined in an area that covers the North Sea and the Baltic Sea. The model consists of three levels of nested bathymetries. A 9 nm (16 668 m) grid covering the North Sea and the Baltic Sea, a 3 nm grid covering the Danish waters and a 1 nm grid of the Danish straits and the western Baltic Sea. The model bathymetry is presented in Figure 1.

The two open boundaries are located in the North Sea. The northern boundary is defined between Wick (Scotland) and Stavanger (Norway) and the southern

TABLE 1. Grid co-ordinates of the measurement (M) and validation (V) stations in the regional model

Area	Station	Type	Grid	Regional x co-ordinate	Regional y co-ordinate
British Coast	Aberdeen	M	9 nm	7·0	43·0
	Lowestoft	M	9 nm	18·0	10·0
	Wick	V	9 nm	5·0	52·0
	Immingham	V	9 nm	12·0	19·0
Danish West Coast	Esbjerg	M	3 nm	44·3	28·0
	Hvide Sande	V	3 nm	43·0	32·7
	Torsminde	V	3 nm	43·0	34·7
	Thyborøn	V	3 nm	43·7	37·3
Kattegat and Skagerak	Hanstholm	M	3 nm	45·3	40·0
	Frederikshavn	M	3 nm	52·3	42·3
	Hirsthals	V	3 nm	50·3	43·3
	Göteborg	V	3 nm	56·3	43·3
	Smogen	V	3 nm	54·3	48·3
Inner Danish Waters	Hornbæk	M	1 nm	59·6	33·3
	København	M	1 nm	60·4	30·6
	Fredercia	M	1 nm	49·7	29·4
	Korsør	M	1 nm	54·7	27·9
	Gedser	M	1 nm	58·1	22·9
	Rødby	V	1 nm	55·8	23·2
	Fynshavn	M	1 nm	50·4	25·8
	Viken	V	1 nm	59·9	33·6
	Slipshavn	V	1 nm	53·7	27·6
	Åbenrå	V	1 nm	48·3	25·9
	Aarhus	V	1 nm	51·3	33·3
Baltic Sea	Tejn	M	1 nm	69·0	28·4
	Kungsholmfort	V	1 nm	71·2	34·1
	Marviken	V	3 nm	74·3	51·0

boundary in the British Channel between Dungeness (UK) and Wissant (France). The water levels at the boundaries are specified at each grid point and for each time step by the sum of the astronomical tidal level and the static atmospheric pressure correction. The astronomical tidal level has been calculated at the northern boundary using the 10 largest tidal constituents and at the southern boundary using the constituents available in the Admiralty Tide Tables.

The meteorological forcing consisted of analysed wind (10 m) and surface air pressure fields covering the simulation period. The meteorological data has been obtained from the operational meteorological model, HIRLAM (Danish Meteorological Institute (DMI)) with a temporal resolution of 6 h and a spatial resolution of 0·21 degrees. The flow resistance for this case is defined with a constant Manning number equal to 32 $m^{1/3}/s$ in the whole model area (i.e. only a rough calibration was used in this model). The model time step has been selected equal to 10 min.

During the simulation period water level data from 27 stations spatially distributed in the model area were available. Table 1 presents the position of the observation points in the regional 9 nm grid, distributed in five different regions: the British coast, the Danish West coast, the Kattegat and Skagerak region, the inner Danish waters and the Baltic sea. Moreover, it is also indicated in Table 1 if the station has been considered either as a measurement station or a validation station in the simulation period. Figure 1 shows the geographical position of the stations in the model domain.

Test Results

The period of study covers a 13-day period from 1st October 1997 00:00 to 14th October 1997 00:00, where water level measurements were available every hour at the 27 stations aforementioned. The measurements were linearly interpolated for assimilation into

TABLE 2. RMSE at all the stations from the deterministic (Det) and the updated model (KF)

Area	Station	Type	RMSE Det (m)	RMSE KF (m)
British Coast	Aberdeen	M	0·34	0·025
	Lowestoft	M	0·23	0·042
	Wick	V	0·24	0·22
	Immingham	V	0·97	0·44
Danish West Coast	Esbjerg	M	0·41	0·055
	Hvide Sande	V	0·21	0·11
	Torsminde	V	0·22	0·089
	Thyborøn	V	0·19	0·099
Kattegat and Skagerak	Hanstholm	M	0·17	0·030
	Frederikshavn	M	0·19	0·022
	Hirsthals	V	0·18	0·076
	Göteborg	V	0·15	0·061
	Smogen	V	0·19	0·078
Inner Danish Waters	Hornbæk	M	0·20	0·026
	København	M	0·23	0·12
	Fredercia	M	0·27	0·038
	Korsør	M	0·26	0·060
	Gedser	M	0·31	0·039
	Rødby	V	0·31	0·050
	Fynshavn	M	0·36	0·041
	Viken	V	0·16	0·10
	Slipshavn	V	0·24	0·053
	Åbenrå	V	0·36	0·062
	Aarhus	V	0·23	0·070
Baltic Sea	Tejn	M	0·17	0·032
	Kungsholmfort	V	0·090	0·058
	Marviken	V	0·054	0·14

TABLE 3. Average results per area in the validation stations

Area	Average RMSE Det (m)	Average RMSE KF (m)	Average STDEV estimated (m)
British Coast	0·61	0·33	0·13
Danish West Coast	0·21	0·099	0·088
Kattegat and Skagerak	0·17	0·072	0·064
Inner Danish waters	0·26	0·067	0·058
Baltic Sea	0·072	0·097	0·094

the model every 20 min in order to ensure that the model is not drifting too far away from the measurements between two assimilation cycles. The model is initialized on 1st October 1997 at 00:00 with water level and velocity fields obtained from a spin-up simulation of 72 h, which is sufficient to develop the main flow conditions in the North Sea and the inner Danish waters. The test case has used a constant Kalman gain generated by an off-line ensemble Kalman filter computation. The constant Kalman gain has been obtained from a 2-day simulation as the average of the Kalman gains during the last 24 h in order to avoid influence from the initialization.

The performance of the system has been evaluated by comparing observations and model results for the 27 available stations. Only 12 stations were assimilated, and therefore it is especially important to study the performance in the locations that were not assimilated (the validation stations). The 12 stations are sufficient to cover the whole area where data are

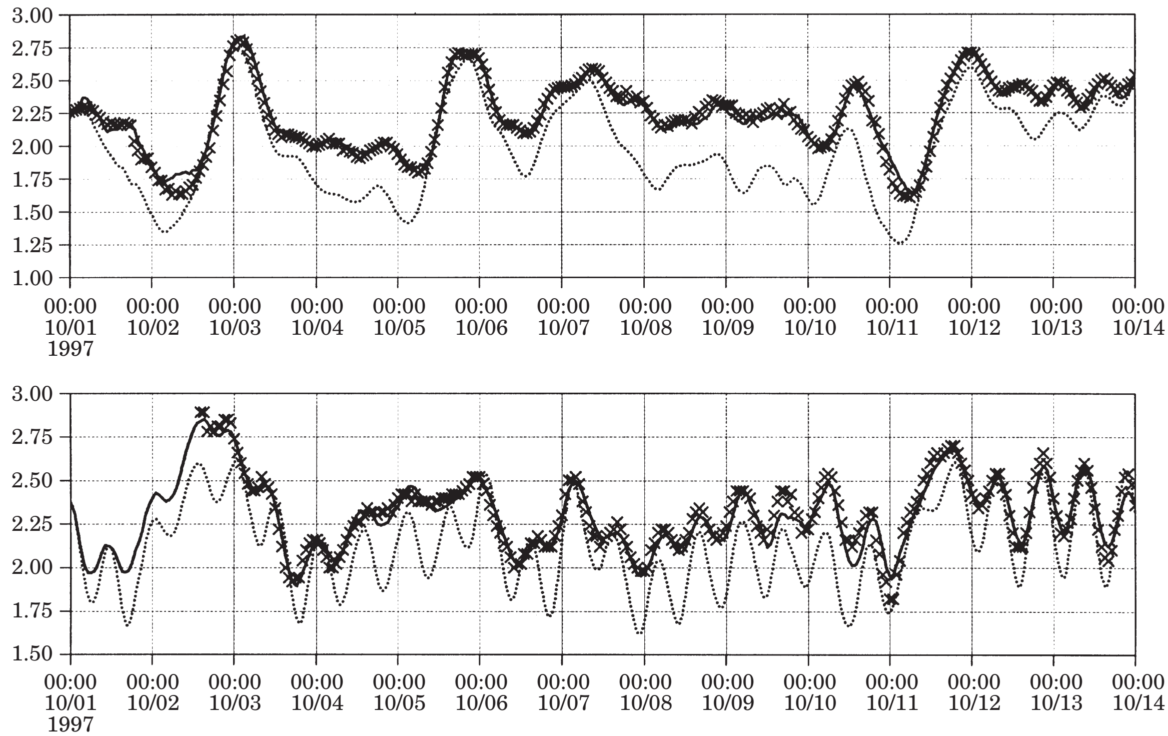

FIGURE 2. Time series of water level from observations (crosses), deterministic model (dotted line) and updated model with the Kalman filter (full line) at two stations in the inner Danish waters. Top: Rødby; Bottom: Slipshavn.

available, avoiding duplication of information. The results can provide an assessment of the capabilities of the system to correct globally the model using only a few observations. For this purpose the root mean square error (RMSE) between the observed and updated water levels are calculated using hourly data for the 13 days of simulation and compared with the RMSE between the observed and the deterministic model simulation (simulation without measurement update). Table 2 presents the RMSE for the deterministic and the updated model with the constant Kalman gain for all the stations. It can be seen that the errors in the updated model are significantly smaller than the errors in the deterministic model. It is important to notice the significant reduction of the error that has been achieved in most of the validation stations.

The performance in the validation stations in a certain area can be considered as an indicator of the filter performance in that area. Table 3 presents the spatial average RMSE of the validation station for each of the different model areas. The error reduction in the first two areas (the British Coast and the Danish West Coast) is in the order of 50%. In the Kattegat and Skagerak area and especially in the inner Danish waters the error reduction is pronounced. The RMSE at the validation stations in the inner Danish waters has been reduced by 75%. Figure 2 shows the time series of water levels at two validation stations (Rødby and Slipshavn) in the inner Danish waters. Three different time series are presented for each station: observed, simulated by the deterministic model and updated using the constant Kalman filter. It can be seen from this figure that the updated model provides an estimation of the water levels very close to the observations.

The error in one of the Baltic Sea stations (Marviken) is larger in the updated model than in the deterministic model, whereas in the other two stations (Tejn and Kungsholmfort) the error has been reduced. It has to be pointed out that Marviken is the most eastern station and that no data has been assimilated in the whole Baltic Sea. Thus, in order to improve the results in the Baltic Sea, it will be necessary to assimilate some data in that region. Another problem is caused by the initial conditions. As it can be observed in Figure 2, there is an off-set between observed and deterministic water levels in the inner Danish water, which is corrected by the Kalman filter. In order to correct this off-set, water from the Baltic Sea area is displaced to the inner Danish waters. The results are improved in the areas of influence of the measurements while they deteriorate in the other parts of the model. This problem is a consequence of

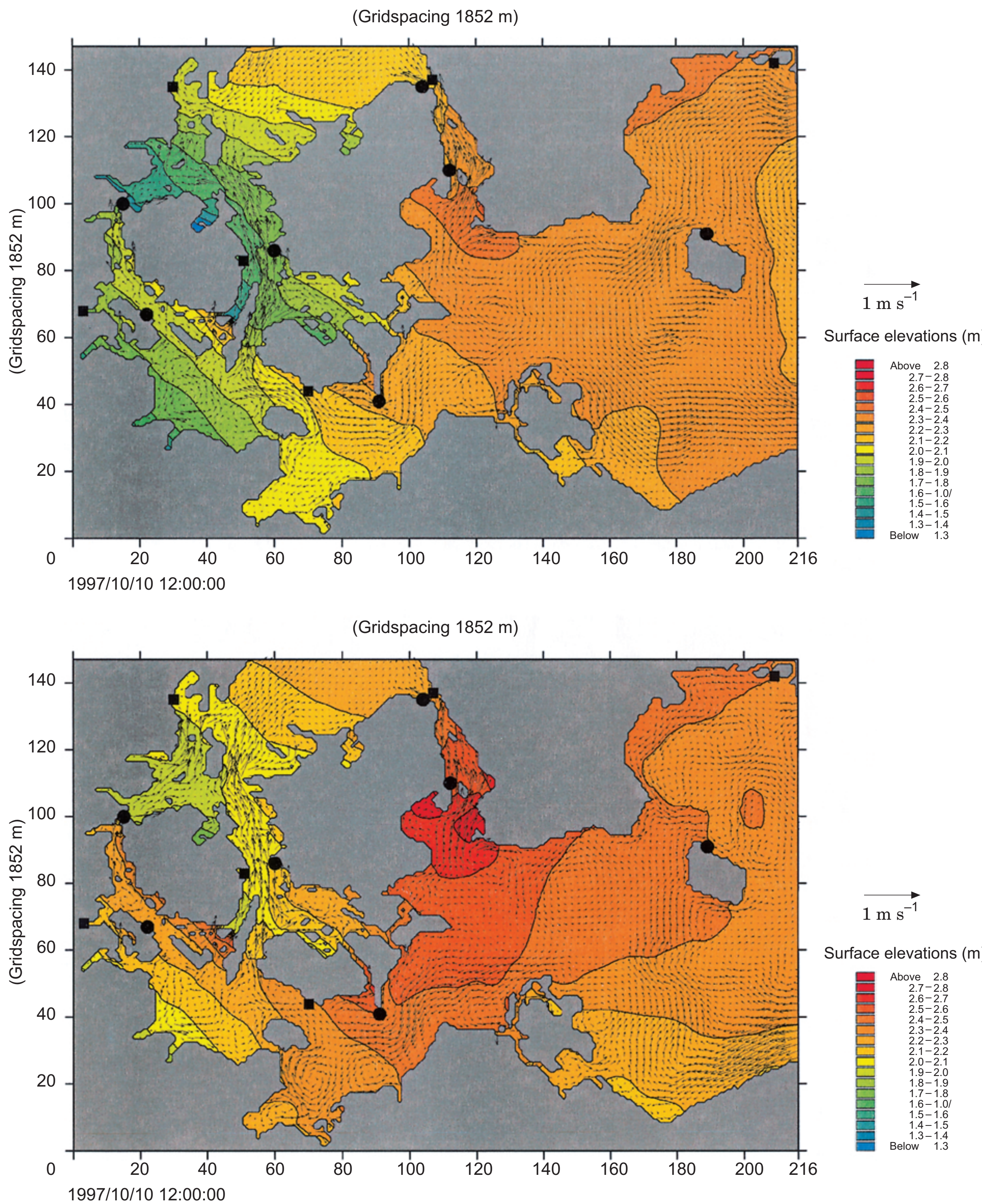

FIGURE 3. Water level and velocity fields on the 10 October 1997 at 12:00 in the inner Danish waters, obtained from the deterministic model (top) and the update model with the Kalman filter (bottom). Circles (measurement) and squares (validation) represent water level stations.

incorrectly specified initial conditions (initial volume). Under the assumption of errors in the open boundaries and the wind stress, the filter can modify the volume of the system only through the boundaries. In this case they have a minor influence in the Baltic Sea area. The problem of initial errors and mass conservative filters is described in more detail in Cañizares (1999).

Figure 3 shows the water level and velocity fields at one specific time step for both the deterministic and the updated model in the inner Danish waters. Significant differences in water levels can be observed between the model results. The updated model results presents higher water levels than the deterministic model. This can also be observed from the time series presented in Figure 2.

As described above, the Kalman gain (weighting matrix) has been obtained from a 2-day simulation as the average of the Kalman gains during the last 24 h. An average error covariance matrix (defined as matrix P in Eq. (5)) has been estimated for the same period. The error covariance matrix contains the estimation of the error for every variable of the state vector. In the last column of Table 3 is shown the average value of the estimated standard deviation of the error in the water levels at the validation stations for each of the model areas. These values represent the Kalman filter estimation of the errors in the water levels. It is seen that these values are very similar to the RMSE of the water level in the updated model, which represents the real error in the system. The filter provides a good estimation of the error in all the areas except in the North Sea where it significantly underestimates the error. In the Baltic Sea area the filter predicts a large error consistent with the observed RMSE.

Conclusions

A prototype of an operational system for storm surge prediction in the Danish waters has been developed. The system consists of three elements. The first element is a network of water level stations distributed in the North Sea, the inner Danish waters and the Baltic Sea. The second element of the system is a two-dimensional shallow water equation model that computes simultaneously areas of different resolution in order to have a better resolution in the inner Danish waters. The last element is a data assimilation method; in this case a Kalman filter with a constant weighting matrix based on an off-line realisation of the ensemble Kalman filter. The fact that the weighting matrix is constant in time avoids the expensive propagation of the error covariance matrix in the Kalman filter, and therefore the method is very suitable for operational systems. The operational model is only slightly more expensive than a model run without data assimilation.

The use of data assimilation provides a corrected solution of the model variables conditioned on the observed water levels. The test results show that the filter is able to efficiently correct the model results in all the areas where the measurements have some influence. This corrected estimate of the state of the system can then be used as initial conditions for forecasting, which is the usual procedure applied in storm surge modelling.

The test results also show that the filter provides an accurate estimation of the error in the system, by means of the error covariance matrix. This estimate provides valuable information about the accuracy of the predictions obtained from the operational system.

Future work will focus on the study of the forecasting capabilities of the operational system. The implementation of this system in a 3D version of the model is already ongoing.

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